Traditional Logic: Subalternation

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An Inky Publication

DEDICATION

To those who believe: All learning is humility before the facts.

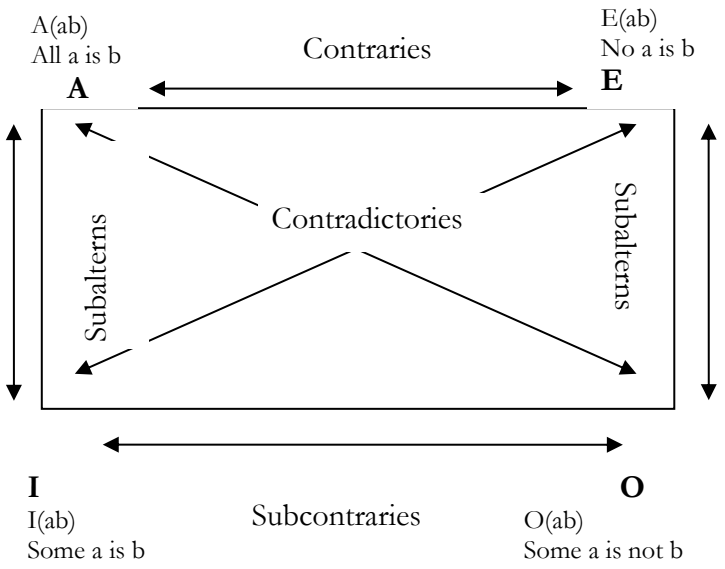
ACKNOWLEDGMENTS

To name all those who have contributed to my study and application of logic whether in classrooms or elsewhere would require an extended essay to acknowledge each fully and accurately. A special note of gratitude though is due to Gordon H. Clark from whom I have learned much about philosophy, theology, and logic. And to my wife and companion, Sharon M. Carranza, a special note of recognition and gratitude. Her labor of patience and concern for clarity and accuracy improved the work substantially.

1 SUBALTERNATION DEFINITION

Subalternation is defined as "one of four kinds of opposition (contradiction, contrariety, subcontrariety, and subalternation); the opposition that exists between propositions alike in quality but different in quantity." (Clark, reference 1, p. 131)

The diagram for the traditional square of opposition is:



In the square of opposition, A and I are subalterns; E and O are subalterns. The subalterns may both be true and may both be false. The truth of the A or E includes the truth of the I or the O, respectively. From the truth of the I or the O, we cannot infer the truth of the A or the E. However, from the falsity of the I or the O, we can infer the falsity of the A or the E, respectively. From the truth of an E, the corresponding A and I are false; from the truth of the A, the corresponding E and O are false. These inferences can be readily ascertained by the use of the Venn diagram, or Euler circles diagram, or the square of opposition.

A	All a is b	Universal Affirmative
E	No a is b	Universal Negative
I	Some a is b	Particular Affirmative
O	Some a is not b	Particular Negative

Two propositions are contradictory if and only if they cannot both be true and they cannot both be false.

- All a is b and Some a is not b are contradictories.

- No a is b and Some a is b are contradictories.

Two propositions are contraries if and only if they cannot both be true but can both be false.

- All a is b and No a is b are contraries.
- Some a is b and Some a is not b are subcontraries.

Two propositions are subcontraries if and only if they cannot both be false but can both be true.

- Some a is b is a subaltern of All a is b.
- Some a is not b is a subaltern of No a is b.

A proposition is a subaltern of another if and only if it must be true if its superaltern is true, and the superaltern must be false if the subaltern is false.

Modern logic reduces the number of valid moods (syllogisms) from 24 to 15 based on how one chooses to define "all". In the proposition, "All men are mortal," according to modern logic, "all" has no existential import. In other words, universal propositions do not imply the existence of the subject, and therefore do not imply the subalternate particular propositions. "All lions in the room next door have sharp teeth" does not imply that "Some lions in the next room have sharp teeth" simply because none exist in the next room. (Douma, Reference 4, p. 196)

Thus according to symbolic or modern logic, "all" in universal propositions does not imply the existence of "some" in particular propositions. The universal has no existential import. In denying existential import, 9 valid syllogisms of traditional logic were declared invalid, leaving only 15. Only the X (contradictories) of the square of opposition remains in modern logic.

Valid Forms or Moods (15)

Figure 1	Figure 2	Figure 3	Figure 4
AAA-1 Barbara EAE-1 Celarent AII-1 Darii EIO-1 Ferio	AEE-2 Camestres EAE-2 Cesare AOO-2 Baroko EIO-2 Festino	AII-3 Datisi IAI-3 Disamis OAO-3 Bokardo EIO-3 Ferison	AEE-4 Camenes IAI-4 Dimaris EIO-4 Fresison
4	4	4	3

The following moods are invalid according to symbolic logic. (9)

Figure 1	Figure 2	Figure 3	Figure 4	Requires that the:
AAI EAO	AEO EAO	(Weakened Forms)	AEO	Subject exists
(Weakened Forms)		AAI EAO	EAO	Middle Term exists
			AAI	Predicate exists
2	2	2	3	Totals: 9

2 CLARK'S ANALYSIS

Modern logic, according to Clark, "has failed to put Aristotelian logic into symbolic form." The source of this failure is modern logic's definition of "all". As Clark notes: "If 'all a is b' means 'a is included in b', subalternation cannot be defended." For example, Russell reduced the sentence, "All Athenians are Greek" not merely to a(ab), but further to (a < b); that is to say, class a is included in class b. If this definition is accepted, modern symbolic logic follows without a hitch. Its conclusions follow necessarily." The result is the reduction of the 24 valid frames of traditional logic to the 15 of modern logic. This outcome is the result of defining "all" in 'all a is b' as (a < b), all of class a is included in class b which thereby denies existential import to universal propositions. (Clark, Reference, 3, p. 23-24; Reference 2, pp. 49-51)

Thus, in modern logic, the question that emerges is whether the inclusion of a class in a higher class will allow or permit the inclusion of an individual in a class. Can "all" imply "some"? According to Clark, there is no question that given modern logic's definition of "all", the assumption of subalternation as valid leads to a false conclusion as he demonstrated in a reductio outlined in what follows. (Clark, Reference 3, pp. 22-23)

Clark's Reductio Argument

His argument should be read in full.

> "One is the class that contains all classes. Zero is the class that contains nothing. Now, since zero is a class, a class used extensively in modern logic, and since 'one' is the class that contains all classes, it follows that (o<i), that is, zero is contained in one. Now try to follow carefully. Since on Russell's view (a<b) defines "all a is b," then on Russell's view (o<i) means "all zero is one." but it cannot mean this. When we say, in English, that "all dogs are animals," we mean that every dog is an animal. Hence, if we say with Russell that all zero is one, we would mean that every zero, every class that contains nothing, is a one, a class that contains all classes. Or to make it still clearer, since zero not only means an empty or null class, but also a false proposition, (o<i) would mean, all false statements are true. Let it be true that zero is included in one: since zero is a

> class, and since one is the class that contains all classes, (o<i) must be true. But it is not the definition of all. Zero is included in one, but it is false to say "all zero is one." Hence Russell's definition of 'all' is faulty, and his completely valid deductions from his faulty definition have nothing to do with *all*, *some*, or subalternation." (Clark, Reference 1, pp. 82-83)

Russell's Blunder

> "I now assert that Russell made a great blunder, not in his deductions but in his definition of the term all. Remember, he explained All a is b as a is included in b. However obvious Russell's definition seems, and it seems so to the immense majority of logicians, it is not a correct analysis of the English word All." (Clark, Reference, 2, p. 51)

In short, Russell's definition of *all* would mean that all false statements are true. (Clark, Reference 1, p. 83 logic) Definitions are not deduced. They are chosen and therefore must be judged by the consequences. One consequence is that in the case of defining *all* as meaning 'a class in a higher class', does not allow inclusion of every individual of a class in a higher class. Modern logic's definition of all fails to put traditional logic into symbolic form. Moreover, its symbolic language cannot say as much as ordinary language can since it fails to express the full meaning of *all* : each and everyone of the members of a class.

3 CLARK'S EXPLANATION

Assuming the following:

(1) zero (0), the null class, is the class of no members, including nothing;
(2) number one (1), the universe (i), is the class that includes everything;
(3) the contradictory of zero is one; zero (0 or o) contains nothing, the universe of discourse contains everything;
(4) (o<i) = the universe contains the null class, or zero (0 or o) is included in one (1 or i); (falsehood, being but another way of expressing the null class, implies truth);
(5) (i<i) & (o<o) = every class implies itself (truth implies truth); and
(6) all = (a< b), a is included in b.

With these assumptions, modern logic's argument that subalternation is a fallacy follows.

Keep in mind:

- A form: All a is b = A(ab) = (a<b), class a is included in class b: definition of ALL.
- E form: No a is b = E(ab') = (a<b'), a is included in contradictory of b (obversion of a form)
- I form: Some a is b = I(ab) = (a< b')', a is not included in contradictory of b (contradictory of e form)
- O form: Some a is not b = O(ab) = (a <b)', it is not the case that a is included in b (contradictory of a form)

The Rejection of Subalternation

1.	[(all a is b) < (some a is b)], or [(a<b) < (a<b')']	Subalternation ∴∠ assuming: RAA, Traditional Logic's Subalternation
2.	(o < o) < (o< o')'	From 1, substitution, if o is included in o, implies o is not included in the contradictory of o.
3.	(o<o)	From 2, true premise since every class is a member of itself; zero is included in zero.
4.	(o<o')'	From 2 & 3, modus ponens, zero not included in the contradictory of zero which is "i".
5.	(o<i)	Definition of universe (i), the class that includes all classes, and from 3, every class is member of itself.
6.	∴(o<o')'	False, from 2-5, o is not included in contradictory of o which is i, the class of all classes, the universe.
7.	[(o< o) < (o<o')']	A true premise and a false conclusion.
Therefore, subalternation is invalid since it leads to a false conclusion.		

> "Since every class is included in itself, (o < o) must be a true statement. But since the universe includes all classes, zero must be included in the universe. Its negation is false. Hence, the last line of the symbolism has a true premise and false conclusion. Therefore ALL does not imply SOME." (Caps, mine; Clark, Reference 1, p. 23)

Thus, subalternation, according to modern logic, cannot be defended. Modern logic does not give us the 24 valid moods of traditional logic; it gives us only 15. "Modern logic has failed to put Aristotelian logic into symbolic form. Its language cannot say as much as ordinary English can." (Clark, Reference 1, p. 23) Modern logic is inferior to traditional logic. It fails to represent accurately ordinary language.

4 OUTLINE OF CLARK'S DEFINITION

Subalternation Outline of Clark's Argument (Reference 1, pp. 81-85)

1. Modern logic denies subalternation and reduces valid moods from 24 valid moods to 15. This is due to Russell's definition of "all" as A(ab) translated (a < b), the whole of class a included in class b.

Clark's definition of All a is b: {(a < b) [(b < a) + (a < b')' (b' < a)']}:*

a) Preserves alternation;

b) Closer to English language;

c) "All a is b" means "every a is b"; and

d) Russell's "all" leads away from serious argumentation.

2. Clark's refutation of Russell's "all" definition

a) One is a class that contains all classes

b) Zero is a class that contains nothing

3. First argument

a) Zero is a class;

b) One is a class that contains all classes; and

c) ∴Zero is contained in one.

2. Second argument:

a) Russell: (a < b) defines "All a is b";

b) 0 and 1 are classes;

c) (0 < 1) which means "all zero is one" ; which is absurd.

3. Otherwise, we mean:

a) Every class that contains nothing is a one, a class that contains all classes; and

b) If zero means a false proposition, (0 < 1) must mean "all false statements are true"

4. If we assume zero is included in one, (0 < 1) is true by this:

- (0 < 1) zero is a class; one is the class containing all classes

  ∴ (0 < 1) must be true, but this not a definition of 'all' because

False to say "all zero is one".

Therefore, Russell definition of "all" is faulty.

Valid deductions have nothing to do with modern logic's rejection of *all*, *some*, or subalternation.

5. Existential import

- Force of argument depends on the symbolism, not the meaning of "existential import";
- Categorical syllogisms are hypothetical; bare logic doesn't assert truth of any premise; and
- Logic as such makes no assertion about existence or nonexistence of anything;
- Therefore, one can dispense with "existential import" and preserve subalternation.

6. Quantification provides no answer to Clark's defense of subalternation

Type of argument used to justify quantification:

All humans are mortals.

Socrates is human.

∴ Socrates is mortal.

Above translated to:

M

S

———

∴ H

Objection is that in the above notation, validity criteria do not apply; therefore, is the form invalid? If validity criteria cannot be applied, perhaps the notation is inadequate.

Clark's Response:

a) Notation is faulty or wrong;

b) Logic alone does not determine the number of objects in a class;

c) "a" in A(ab) can be a single object;

d) The argument about Socrates above is Barbara (AAA-1);

e) Singular propositions assumed to be totally different from universal propositions;

f) 'e)' above, led to introduction of theory of quantification and complicated formulas;

g) But singular propositions are simply ordinary universal propositions.

> "But contemporary logic thinks that singular propositions are something entirely different from universal propositions, and that therefore they cannot be handled by Aristotelian methods. To remedy this alleged defect, the theory of Quantification invents rather complicated formulas. To say the least, this expenditure of ingenuity is unnecessary. When we speak of a class, which has only one member, we are speaking about all of that class. Thus a so-called singular proposition is simply an ordinary universal." (Clark, Reference 1, p.85)

5 SUMMARY AND REVIEW

Modern logic defines 'All a is b' as (a < b) or (o < i) where "o" is zero and "i" is the universe, the class that contains all classes. Clark's analysis reveals that the definition of *all*, as such, fails on several counts:

1. Definitions are chosen and to choose to define *all* as (a < b) fails to symbolize all of the valid moods of traditional logic, reducing 24 to 15 valid moods.

2. The definition of All as (a < b) means that the null class is included in the class that includes all classes (o < i). But this definition of *all* promotes the invalid conclusion that "every zero is a one" violating the law contradiction. Therefore, this definition of *all* is false, since it yields a contradiction; zero and one are contradictories. (Clark, Reference 2, p. 51)

3. Existential import is the idea that a universal like 'All a is b' does not assert the existence of *any* a or b. However, 'Some a is b' asserts the existence of at least one a. In other words, *all* does not imply *some* without qualification.

> "Hence, since there is an existential factor in I(ab), and none in A(ab), and since a valid inference cannot have a factor in the conclusion which was absent from the premises, A(ab) < I(ab) must be invalid. But if we insist, these modern logicians must acknowledge that 'existential import' is a phrase in ordinary (though not colloquial) English; and that, therefore, whatever force their arguments have must depend on the symbolism. Talk about existential import therefore is irrelevant." (Clark, Reference 1, p. 83)

> "Furthermore, ordinary English, or ordinary logic, has no room for existential import. … Logic alone does not assert the existence or non-existence of anything. It would seem, therefore, that we can dispense with existential import and preserve the validity of subalternation." (Clark, Reference 1, pp. 83-84)

In short, modern logic asserts that traditional logic's definition of "all" means that *all* implies *some*. The argument is: 'all lions in my bedroom are tame,' cannot imply that 'some lions in my bedroom are tame', simply because there are no such lions in my bedroom. The subject term does not exist.

One could say, 'if there were lions in my bedroom, then these lions would be tame lions.' this states a condition that may or may not obtain. Thus, an A-form cannot imply the corresponding I-form, it is argued, because the existence of the subject term in the A-form may or may not be the case. However, logic does not determine the existence or non-existence of any subject term or any predicate term of any of the forms of traditional logic or any logic. Thus, the 'all' of traditional logic means 'each and every member of a class' whether or not such members exists. Logic is the science of necessary inference. It neither asserts existence nor non-existence of the terms in premises and conclusions.

> "Russell explained that 'all men are mortal' means 'if anything is a man, it is mortal.' it does not mean that there are any men. However, the phrase, 'some men are mortal' means 'there exists at least one man and he is mortal.' hence if all men are mortal, it does not follow that some are. What has happened is that the symbolism invented between 1850 and 1900, although it preserved contradiction and obversion, made subalternation a fallacy." (Clark, Reference 3, p. 22)

Traditional logic is to be preferred for these reasons.

> "Note especially the conclusion Clark reaches regarding Russell's view. It would mean all false statements are true. It shows why Russell's view presents 'no possibility of coherent thought or intelligible discourse.' Thus, Russell's view cannot compete with traditional logic. There is only one valid logic known, and that is Aristotle's traditional logic." (Douma, Reference 4, p. 198 & Clark, Reference 3, p. 24)**

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*Modern logic denies the following moods of traditional logic: Darii, Darati, Felapton, Fesapo, and Bramantip (AAA-4). Also, AAI-1; EAO-1; EAO-2; AEO-2; AEO-4. (Bramantip is AAI-4 and AAI -1)

**Clark's definition of "all", not without some difficulty but still adequate for traditional logic, can be found in his book, Logic, pp. 81-85. (See also Carranza, E. Logic Workbook for Clark's Logic, pp. 55-57)

"{(a<b) [(b<a) + (a<b')' (b'<a)']}

(a<b)	[(b<a)	(a<b')'	(b'<a)'
1	2	3	4

1. (a is included in b) &

2. EITHER, (b is included in a)

3. OR, (it is not the case that a is included in the contradictory of b) AND

4. It is not the case that the contradictory of b is included in a.

Therefore, All a is b."

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